

Finite Temperature Excitations of an Inhomogeneous Trapped Bose Gas With Feshbach Resonances*

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We derive and discuss the temperature dependance of the condensate and noncondensate density profile of a Bose-Einstein condensate gas with Feshbach resonance in a parabolic trap. These quantities are calculated self-consistently using the generalized Hartree-Fock-Bogoliubov (HFB) equations within the Bogoliubov approximation. At zero temperature, the HFB equation can be solved by means of a variation method that give the low excitation spectrum. Moreover, within the two-body collision theory, we estimate the relationship between the atom number and the external magnetic field B , it is in good agreement with the data in recent experiments.

Bose-Einstein condensates[1-4] of atomic gases offer new opportunities for studying quantum-degenerate fluids. Almost all the essential properties of Bose-Einstein condensate systems such as the formation and shape of the condensate and noncondensate, the nature of its collective excitations and statistical fluctuations, and the formation and dynamics of vortices are determined by the strength of atom-atom interactions. In contrast with the situation in traditional superfluids, the strength of inter-particle interactions in the atomic condensate can vary over a wide range of values[5-12]. In particular, the scattering length that characterizes the atom-atom interactions can be negative, corresponding to an effective inter-atom attraction.

Most recently, in trapped atomic Bose-Einstein condensation, Ketterle's group reported evidences for modifying the scattering length by magnetic-field-induced Feshbach resonance[13]. Feshbach resonance have been studied twenty one years ago[14] at much higher energies, but the Feshbach resonance energy observed in ultracold atoms can be tuned to near zero. The theoretical studies of the ultracold atoms with Feshbach resonance[15,16] show that the two-body interactions responsible for the Feshbach resonance produce an additional condensate of molecules[17], which differs qualitatively from the properties of a single condensate. In this letter, we discuss the temperature dependance of the hybrid atomic/molecular condensate and noncondensate density profile as well as the excitation spectrum. Especially, for an inhomogeneous system with negative scattering length, the excitation spectrum shows a upper constraint on the atom number, which, together with the conventional estimation of the condensate atom number, gives the relationship between the atom number and the external magnetic field B . The theoretical results are in good agreement with the experiment.

The binary atom Feshbach resonances studied by varying a strong external magnetic field in an alkali-atom trap are hyperfine-induced spin-flip processes that bring the colliding atoms to a bound molecular state of different electron spin. This process can be described by the Hamiltonian

$$H_{FR} = \alpha \int d^3r \Psi_m^+(r) \Psi_a(r) \Psi_a(r) + h.c., \quad (1)$$

where $\Psi_m(r)$, $\Psi_m^+(r)$ ($\Psi_a(r)$, $\Psi_a^+(r)$) are the annihilation and creation field operators of the molecules(atoms), α stands for the transition matrix element proportional to the overlap of the molecular continuum and bound state wave functions. Usually, the transition matrix element depends on the magnetic field as $\alpha \sim \sqrt{\lambda_a \Delta^2 / 2|B - B_0|}$, where B_0 is the resonant magnetic field and Δ characterizes the width of the resonance as a function of B . The Hamiltonian H_{FR} together with the atomic Hamiltonian

$$H_a = \int d^3r \Psi_a^+(r) \left[-\frac{\nabla^2}{2M} + V_a(r) - \mu_a \right] \Psi_a(r) + \frac{\lambda_a}{2} \int d^3r \Psi_a^+(r) \Psi_a^+(r) \Psi_a(r) \Psi_a(r), \quad (2)$$

the molecular Hamiltonian

$$H_m = \int d^3r \Psi_m^+(r) \left[-\frac{\nabla^2}{4M} + V_m(r) - \mu_m + \epsilon \right] \Psi_m(r) + \frac{\lambda_m}{2} \int d^3r \Psi_m^+(r) \Psi_m^+(r) \Psi_m(r) \Psi_m(r), \quad (3)$$

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and the atom-molecule interaction Hamiltonian

$$H_{am} = \lambda \int d^3r \Psi_a^+(r) \Psi_m^+(r) \Psi_m(r) \Psi_a(r) \quad (4)$$

forms a total Hamiltonian $H = H_a + H_m + H_{am} + H_{FR}$, which governs the dynamics of the system under investigation. Here, $V_{a(m)}(r)$ represents the trapped potential for atom (molecule), $\mu_{a(m)}$ is the chemical potential of the atoms(molecules), $\lambda_{a(m)} = \frac{4\pi a_{a(m)}}{(2)M}$, with M being the atomic mass and $a_{a(m)}$ the s-wave scattering length of the atom-atom interaction, λ denotes the coupling constant of the atom-molecule interaction, and ϵ is the energy of the intermediate molecular state relative to the continuum of the incident atoms.

Separating out the condensate part in the usual fashion (Bogoliubov approximation) i.e.

$$\Psi_{a(m)}(r) = \phi_{a(m)}(r) + \hat{\varphi}_{a(m)}(r), \quad (5)$$

where $\phi_{a(m)}(r) = \langle \Psi_{a(m)}(r) \rangle$ plays the role of a spatially varying macroscopic Bose field of the atoms(molecules). The possibility that the resonantly formed quasibound atom pairs form a molecular condensate was previously suggested by Timmermans *et al.*[15]. Using a Raman photonassociation process, the quasibound pairs start to be formed from the atomic condensate and form a molecular condensate[17]. Here, we assume that there are a large number of atoms and molecules in the condensate. It is easy to show that the operators $\hat{\varphi}_{a(m)}(r)$ and $\hat{\varphi}_{a(m)}^+(r)$ obey the Bose commutation relations

$$[\hat{\varphi}_{a(m)}(r), \hat{\varphi}_{a(m)}^+(r')] = \delta(r - r'). \quad (6)$$

In terms of $\hat{\varphi}_{a(m)}$ and $\phi_{a(m)}$, the Hamiltonian can be expanded as

$$\begin{aligned} H &= H_0 + H', \\ H_0 &= \int d^3r \{ \phi_a^* \left(-\frac{\nabla^2}{2M} - \mu_a + V_a(r) \right) \phi_a + \frac{\lambda_a}{2} \phi_a^* \phi_a^* \phi_a \phi_a \\ &\quad + \phi_m^* \left(-\frac{\nabla^2}{4M} - \mu_m + V_m(r) + \epsilon \right) \phi_m + \frac{\lambda_m}{2} \phi_m^* \phi_m^* \phi_m \phi_m \\ &\quad + \lambda \phi_a^* \phi_a \phi_m^* \phi_m + \alpha \phi_m^* \phi_a \phi_a + \alpha \phi_m \phi_a^* \phi_a^* \} \end{aligned} \quad (7)$$

$$\begin{aligned} H' &= \int d^3r \{ \hat{\varphi}_a^+ \left(-\frac{\nabla^2}{2M} - \mu_a + V_a(r) \right) \hat{\varphi}_a + 2\lambda_a \hat{\varphi}_a^+ \hat{\varphi}_a \phi_a^* \phi_a + \frac{\lambda_a}{2} (\hat{\varphi}_a^+ \hat{\varphi}_a^+ \phi_a \phi_a + \hat{\varphi}_a \hat{\varphi}_a \phi_a^* \phi_a^*) \\ &\quad + \hat{\varphi}_m^+ \left(-\frac{\nabla^2}{4M} + \epsilon - \mu_m + V_m(r) \right) \hat{\varphi}_m + 2\lambda_m \hat{\varphi}_m^+ \hat{\varphi}_m \phi_m^* \phi_m + \frac{\lambda_m}{2} (\hat{\varphi}_m^+ \hat{\varphi}_m^+ \phi_m \phi_m + \hat{\varphi}_m \hat{\varphi}_m \phi_m^* \phi_m^*) \\ &\quad + \lambda \hat{\varphi}_a^+ \hat{\varphi}_a \phi_m^* \phi_m + \lambda \hat{\varphi}_m^+ \hat{\varphi}_m \phi_a^* \phi_a + \alpha (\hat{\varphi}_a \hat{\varphi}_a \phi_m^* + \hat{\varphi}_a^+ \hat{\varphi}_a^+ \phi_m) \}. \end{aligned} \quad (8)$$

In derivation of eq.(7,8), the following coupling equations are used[15],

$$\begin{aligned} \left\{ -\frac{\nabla^2}{2M} + \lambda_a |\phi_a|^2 + V_a(r) + \lambda |\phi_m|^2 \right\} \phi_a + 2\alpha \phi_m \phi_a^* &= \mu_a \phi_a, \\ \left\{ -\frac{\nabla^2}{4M} + \lambda_m |\phi_m|^2 + V_m(r) + \epsilon + \lambda |\phi_a|^2 \right\} \phi_m + \alpha \phi_a \phi_a &= \mu_m \phi_m. \end{aligned} \quad (9)$$

This coupling equations may be yielded by the expectation value of the Heisenberg equations

$$i\hbar \dot{\Psi}_a = [\Psi_a, H], i\hbar \dot{\Psi}_m = [\Psi_m, H], \quad (10)$$

and replacing the time derivatives by the chemical potentials

$$i\hbar \dot{\phi}_a \rightarrow \mu_a \phi_a, i\hbar \dot{\phi}_m \rightarrow \mu_m \phi_m.$$

The chemical potential of the molecules is twice the chemical potential of the atoms, in accordance with the condition for chemical equilibrium. The α -terms that couple the equations describe tunneling of pairs of atoms between ϕ_m and

ϕ_a fields, it leads to the form of a second condensate—molecular condensate in an atomic Bose-Einstein condensate[15-17]. Using the coupling equations(9), Timmermans *et al.*[15] investigate the behaviors of the hybrid atomic/molecular condensates near- and off- resonance. The Hamiltonian (8) can be diagonalized by using the Bogoliubov transformation

$$\begin{aligned}\hat{\varphi}_a(r) &= \sum_j [u_j(r)\alpha_j - v_j^*(r)\alpha_j^+], \\ \hat{\varphi}_a^+(r) &= \sum_j [u_j^*(r)\alpha_j^+ - v_j(r)\alpha_j], \\ \hat{\varphi}_m(r) &= \sum_j [x_j(r)\beta_j - y_j^*(r)\beta_j^+], \\ \hat{\varphi}_m^+(r) &= \sum_j [x_j^*(r)\beta_j^+ - y_j(r)\beta_j],\end{aligned}\tag{11}$$

where the quasiparticle operators $\alpha_j, \alpha_j^+, \beta_j, \beta_j^+$ obey boson commutation relations

$$[\alpha_i, \alpha_j^+] = \delta_{ij}, [\alpha_i, \alpha_j] = [\alpha_i^+, \alpha_j^+] = 0,$$

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$$[\alpha_i^+, \beta_j^+] = [\alpha_i, \beta_j] = [\alpha_i, \beta_j^+] = 0,$$

and $u_j(r), v_j(r), x_j(r), y_j(r)$ are *c*-number functions. Substituting eq.(11) into eq.(8), one yields

$$H' = \sum_j E_j \alpha_j^+ \alpha_j + \sum_i e_i \beta_i^+ \beta_i - \sum_j E_j \int d^3r |v_j(r)|^2 - \sum_i e_i \int d^3r |y_i(r)|^2\tag{12}$$

with

$$\begin{aligned}(-\frac{\nabla^2}{2M} + \lambda|\phi_m|^2 + 2\lambda_a|\phi_a|^2 - \mu_a + V_a(r))u_j - (\lambda_a\phi_a^*\phi_a^* + 2\alpha\phi_m^*)v_j &= E_j u_j, \\ (-\frac{\nabla^2}{2M} + \lambda|\phi_m|^2 + 2\lambda_a|\phi_a|^2 - \mu_a + V_a(r))v_j - (\lambda_a\phi_a^*\phi_a^* + 2\alpha\phi_m^*)u_j &= -E_j v_j, \\ (-\frac{\nabla^2}{4M} + \epsilon + \lambda|\phi_a|^2 + 2\lambda_m|\phi_m|^2 - \mu_m + V_m(r))x_j - \lambda_m\phi_m^*\phi_m^*y_j &= e_j x_j, \\ (-\frac{\nabla^2}{4M} + \epsilon + \lambda|\phi_a|^2 + 2\lambda_m|\phi_m|^2 - \mu_m + V_m(r))y_j - \lambda_m\phi_m^*\phi_m^*x_j &= -e_j y_j.\end{aligned}\tag{13}$$

In order to study the temperature dependence of the excitation spectrum as well as the spatial distribution of the hybrid atom/molecular condensate and noncondensate, we need to solve the coupled mean-field Bogoliubov equations(13), and the condensate equation(9) self-consistently. The calculations procedure can be summarized for an arbitrary confining potential as follows: First of all, we solve eq.(9) self-consistently, once ϕ_a and ϕ_m are known, the solution of u_j, v_j, x_j and y_j can be generated. To illustrate this procedure, we present its first step of calculations analytically. The trapped potential considered here is taken to be an isotropic harmonic potential $V_{a(m)}(r) = \frac{1}{2}M\omega_{a(m)}^2 r^2$, for which ϕ_a and ϕ_m are spherically symmetric functions,

$$\phi_{a(m)}(r) = R_{00}(r)Y_{00}(\theta, \psi),\tag{14}$$

with

$$R_{00}(r) = \alpha^{3/2} \sqrt{\frac{4}{\pi}} \exp[-\frac{1}{2}\alpha^2 r^2], Y_{00}(\theta, \psi) = \frac{1}{\sqrt{4\pi}}, \alpha = ((2)M\omega)^{1/2}.$$

Rather than solving the coupled equations (13) directly, we introduce a new method based on the auxiliary functions

$$\begin{aligned}u_j &= A_j \langle r|j \rangle_a, v_j = B_j \langle r|j \rangle_a, \\ x_j &= C_j \langle r|j \rangle_m, y_j = D_j \langle r|j \rangle_m,\end{aligned}\tag{15}$$

where $|j\rangle_{a(m)}$ is defined by

$$[-\frac{\nabla^2}{2(4)M} + V_{a(m)}(r)]|j\rangle_{a(m)} = \hbar\omega_{a(m)}(j + \frac{1}{2})|j\rangle_{a(m)}.$$

The reason for such selection is that the level shifts caused by atom-atom interactions weakly depend on the shape of the wave function. A combination of Eqs.(13-15) gives

$$\begin{aligned} (\hbar\omega_a(j + \frac{1}{2}) + \lambda|\phi_m|^2 + 2\lambda_a|\phi_a|^2 - \mu_a)A_j - (\lambda_a\phi_a^*\phi_a^* + 2\alpha\phi_m^*)B_j &= E_j A_j, \\ (\hbar\omega_a(j + \frac{1}{2}) + \lambda|\phi_m|^2 + 2\lambda_a|\phi_a|^2 - \mu_a)B_j - (\lambda_a\phi_a^*\phi_a^* + 2\alpha\phi_m^*)A_j &= -E_j B_j, \\ (\hbar\omega_m(j + \frac{1}{2}) + \epsilon + \lambda|\phi_a|^2 + 2\lambda_m|\phi_m|^2 - \mu_m)C_j - \lambda_m\phi_m^*\phi_m^*D_j &= e_j C_j, \\ (\hbar\omega_m(j + \frac{1}{2}) + \epsilon + \lambda|\phi_a|^2 + 2\lambda_m|\phi_m|^2 - \mu_m)D_j - \lambda_m\phi_m^*\phi_m^*C_j &= -e_j D_j, \end{aligned} \quad (16)$$

the eigenfunctions and the corresponding eigenvalues are given by

$$\begin{aligned} B_j^\pm &= [\frac{1}{f^\pm(r, j) - 1}]^{\frac{1}{2}}, A_j^\pm(r) = f^\pm(r, j)B_j^\pm, \\ D_j^\pm &= [\frac{1}{g^\pm(r, j) - 1}]^{\frac{1}{2}}, B_j^\pm(r) = g^\pm(r, j)D_j^\pm, \end{aligned} \quad (17)$$

and

$$\begin{aligned} E_j^\pm(r) &= \pm\{(\lambda_a\phi_a^*\phi_a^* + 2\alpha\phi_m^*) - [\hbar\omega_a(j + \frac{1}{2}) - \mu_a + \lambda|\phi_m|^2 + 2\lambda_a|\phi_a|^2]\}, \\ e_j^\pm(r) &= \pm\{\lambda_m\phi_m^*\phi_m^* - [\hbar\omega_m(j + \frac{1}{2}) - \mu_m + \epsilon + 2\lambda_m|\phi_m|^2]\}. \end{aligned} \quad (18)$$

Here,

$$f^\pm(r, j) = \frac{\lambda_a\phi_a^*\phi_a^* + 2\alpha\phi_m^*}{\hbar\omega_a(j + \frac{1}{2}) - \mu_a + \lambda|\phi_m|^2 + 2\lambda_a|\phi_a|^2 - (E_j^\pm)^2},$$

and

$$g^\pm(r, j) = \frac{\lambda_m\phi_m^*\phi_m^*}{\hbar\omega_m(j + \frac{1}{2}) - \mu_m + \epsilon + 2\lambda_m|\phi_m|^2 - (e_j^\pm)^2}.$$

These explicit solutions enable us to construct the one-body density matrix

$$\begin{aligned} \rho(r, r') &= \rho_a(r, r') + 2\rho_m(r, r'), \\ \rho_a(r, r') &= \phi_a^*(r)\phi_a(r') \\ &+ \sum_{p=\pm, i=1}^{\infty} [u_i^{p*}(r)u_i^p(r')F_i^p + v_i^{p*}(r)v_i^p(r')(1 + F_i^p)], \\ \rho_m(r, r') &= \phi_m^*(r)\phi_m(r') \\ &+ \sum_{p=\pm, i=1}^{\infty} [x_i^{p*}(r)x_i^p(r')f_i^p + y_i^{p*}(r)y_i^p(r')(1 + f_i^p)], \end{aligned} \quad (19)$$

where $F_i^p = \frac{1}{\exp(\beta E_i^p) - 1}$ and $f_i^p = \frac{1}{\exp(\beta e_i^p) - 1}$ are the Bose distribution for the quasiparticle excitations with energies E_i^p and e_i^p , respectively. Setting $r = r'$, eq.(19) follows the resulting particle density.

We need to point out that eqs. (17) and (18) are results of the first step of the numerical calculations. To complete numerical calculations, we should repeat the above procedures until the eigenvalues E_j and e_j do not depend on position r . In what follows, we present a variation method to study the excitations at zero temperature. This method

was first introduced in Ref. [18] to study the BEC ground state in the harmonic trap of boson system, and it was generalized in Ref. [19] to investigate the excited states in BEC. Considering eq.(13) as well as

$$\int [u_j(r)u_j^*(r) - v_j(r)v_j^*(r)]dr = 1,$$

and

$$\int [x_j(r)x_j^*(r) - y_j(r)y_j^*(r)]dr = 1,$$

which were derived from the Bose commutation relation (6) we arrive at

$$\begin{aligned} E_j &= \int u_j^*(r) \left(-\frac{\nabla^2}{2M} + \lambda|\phi_m|^2 + 2\lambda_a|\phi_a|^2 + V_a(r) \right) u_j dr \\ &\quad + \int v_j^*(r) \left(-\frac{\nabla^2}{2M} + \lambda|\phi_m|^2 + 2\lambda_a|\phi_a|^2 + V_a(r) \right) v_j dr \\ &\quad - \int u_j^*(r) (\lambda_a \phi_a^* \phi_a^* + 2\alpha \phi_m^*) v_j(r) dr - \int v_j^*(r) (\lambda_a \phi_a^* \phi_a^* + 2\alpha \phi_m^*) u_j(r) dr, \end{aligned} \quad (20)$$

$$\begin{aligned} e_j &= \int x_j^*(r) \left(-\frac{\nabla^2}{4M} + \varepsilon + \lambda|\phi_a|^2 + 2\lambda_m|\phi_a|^2 + V_m(r) \right) x_j dr \\ &\quad + \int y_j^*(r) \left(-\frac{\nabla^2}{4M} + \lambda|\phi_a|^2 + \varepsilon + 2\lambda_m|\phi_m|^2 + V_m(r) \right) y_j dr \\ &\quad - \int x_j^*(r) \lambda_m \phi_m^* \phi_m^* y_j(r) dr - \int y_j^*(r) \lambda_m \phi_m^* \phi_m^* x_j(r) dr. \end{aligned} \quad (21)$$

For simplicity, we study only the case of the spherical harmonic trap. In this case, we may choose the trial wave functions of the excitation components $u_j(r), v_j(r), x_j(r)$ and $y_j(r)$ in the form of the spherical harmonic oscillator wave function $\xi_{n_r, l, m}$ with quantum numbers (n_r, l, m) :

$$\begin{aligned} \begin{pmatrix} u_j(r) \\ v_j^*(r) \end{pmatrix} &= \begin{pmatrix} u \\ v^* \end{pmatrix} \xi_{n_r, l, m}(\omega_{n_r l m}, r), \\ \begin{pmatrix} x_j(r) \\ y_j^*(r) \end{pmatrix} &= \begin{pmatrix} x \\ y^* \end{pmatrix} \xi_{n_r, l, m}(\omega_{n_r l m}, r), \end{aligned} \quad (22)$$

where $\omega_{n_r l m}$ is an adjustable scaling factor of variation. Eqs (20) and (21) show that E_j and e_j take a similar form, hence we here discuss branches E_j of the excitation spectra in detail. For $(n_r, l, m) = (0, 1, 0)$, we have

$$\xi_{0,1,0} = \alpha_{010}^{3/2} \left[\frac{8}{3\sqrt{\pi}} \right]^{1/2} \alpha_{010} r e^{-\alpha_{010}^2 r^2/2} Y_{1,0}(\theta, \psi).$$

The excitation spectrum in this case is reduced to

$$\begin{aligned} E &= E[v, \omega_{010}] = (1 + 2v^2) \left[\frac{5}{4} \hbar \omega_{010} + \frac{5}{4} \hbar \frac{\omega_a^2}{\omega_{010}} \right] \\ &\quad + \lambda(1 + 2v^2) N_m \omega_m^{3/2} \left[\frac{2M}{\pi\hbar} \right]^{3/2} \left[\frac{\omega_{010}}{\omega_{010} + 2\omega_m} \right]^{5/2} \\ &\quad + [2\lambda_a(1 + 2v^2) - 2\lambda_a v \sqrt{1 + v^2}] N_a \omega_a^{3/2} \left[\frac{M}{\pi\hbar} \right]^{3/2} \left[\frac{\omega_{010}}{\omega_{010} + \omega_a} \right]^{5/2} \\ &\quad - 4\alpha v \sqrt{1 + v^2} \left[\frac{\omega_{010}}{\omega_{010} + \omega_m} \right]^{5/2} N_m^{1/2} \omega_m^{3/4} \left[\frac{2M}{\pi\hbar} \right]^{3/4}, \end{aligned} \quad (23)$$

where $\alpha_{010}^2 = \frac{M\omega_{010}}{\hbar}$. Similarly, for $(n_r, l, m) = (1, 0, 0)$, we have

$$\xi_{1,0,0} = \alpha_{100}^{3/2} \left[\frac{8}{3\sqrt{\pi}} \right]^{1/2} \left(\frac{3}{2} - \alpha_{100}^2 r^2 \right) r e^{-\alpha_{100}^2 r^2/2} Y_{0,0}(\theta, \psi).$$

And

$$\begin{aligned}
E = E[v, \omega_{100}] = & (1 + 2v^2) \left[\frac{7}{4} \hbar \omega_{100} + \frac{7}{4} \hbar \frac{\omega_a^2}{\omega_{100}} \right] \\
& + \lambda (1 + 2v^2) N_m \omega_m^{3/2} \left[\frac{2M}{\pi \hbar} \right]^{3/2} f(\omega_{100}, 2\omega_m) \\
& + [2\lambda_a (1 + 2v^2) - 2\lambda_a v \sqrt{1 + v^2}] N_a \omega_a^{3/2} \left[\frac{M}{\pi \hbar} \right]^{3/2} f(\omega_{100}, \omega_a) \\
& - 4\alpha v \sqrt{1 + v^2} f(\omega_{100}, \omega_m) N_m^{1/2} \omega_m^{3/4} \left[\frac{2M}{\pi \hbar} \right]^{3/4},
\end{aligned} \tag{24}$$

where

$$f(x, y) = \frac{3}{2} \left(\frac{x}{x+y} \right)^{3/2} - 3 \left(\frac{x}{x+y} \right)^{5/2} + \frac{5}{2} \left(\frac{x}{x+y} \right)^{7/2}.$$

Minimizing the energies of eqs (23) and (24) with respect to the variation parameters v , ω_{010} and ω_{100} , we can determine the excitation spectrum for the mode $(0, 1, 0)$ and $(1, 0, 0)$. The numerical results are illustrated in fig.1 and fig.2. The dashed lines in figures show the excitation spectrum in atom BEC, i.e., $\alpha = \lambda = 0$. In contrast, the solid line are those of hybrid atomic/molecular condensates near Feshbach resonance. From fig.1 and fig.2 we see that while the excitation frequency for modes $(0, 1, 0)$ increases due to the Feshbach resonance effect, the excitation frequency for the mode $(1, 0, 0)$ decreases. We would like to point out that the numerical results presented here depend on the coupling constant as well as the parameter α . In Fig.1 and Fig.2, we let $\alpha = 5\lambda_a$, and $\lambda_a = 0.1$ (arbitrary units). The other parameters in Fig.1 and Fig.2 are $\omega_m = 1.4\omega_a = 7500\text{Hz}$, $N_a = N_m = 10^6$.

For clarity, we illustrate the above somewhat formal discussion by considering the binary atom system for a uniform system ($V_{a(m)}(r) \rightarrow 0$), in this case H_{FR} gives a resonant contribution to the atom-atom interaction strength a_a : $a_{eff} = a_0(1 + \frac{\Delta}{B_0 - B})$, where a_0 is the off-resonant scattering length, and Δ characterizes the width of the resonance. For small p , these excitations are phonons, and their energy tends to zero with p . Hence,

$$\mu_a = \lambda |\phi_m|^2 + \lambda_a |\phi_a|^2 - 2\alpha \phi_m^*, \tag{25}$$

which leads to

$$E_j^2 = E^2(p) = \left(\frac{\hbar^2}{2m} \right)^2 p^2 (p^2 + 16\pi n a_{eff}). \tag{26}$$

For a uniform dilute Bose gas with negative scattering length a_{eff} , eq.(26) implies an instability of those modes with $p^2 \leq 16\pi n |a_{eff}|$. For a gas in a trap, however, the wavenumber cannot be arbitrarily small, and the minimum value is of order $p_{min} \simeq \pi/R_0$ (R_0 is the mean size of the ground state). Hence the system can remain stable if $\frac{\pi^2}{R_0^2} \geq 16\pi n |a_{eff}|$. Since the density is of order $n \simeq N/R_0^2$, this means that the critical number of the system is

$$N_0 \simeq \frac{\pi}{16} \frac{R_0}{|a_{eff}|}. \tag{27}$$

For a positive scattering length a_{eff} , however, there are not any constraints in N . The Bogoliubov quasiparticle theory shows that the condensate atoms N_0 depends on the scattering length and satisfies (for $(a_{eff} \frac{N}{V})^{\frac{1}{3}} \ll 1$).

$$N_0 = N \left(1 - \frac{8}{3} \sqrt{\frac{N a_{eff}^3}{\pi V}} \right). \tag{28}$$

The numerical results of eqs.(27) and (28) are illustrated in Fig.3, which shows the atom number N_0 vs. external magnetic field B . The parameters in Fig.3 are $N/V = N/R^2 = 10^{15}/\text{cm}^3$, $\Delta = 0.01\text{mT}$.

To sum up, we have derived a set of four coupled equations of the atomic and molecular excitations within standard Hartree-Fock-Bogoliubov approximation. As shown in eqs. (9) and (13), the α terms describing the process that converts atoms into molecules play an important role in atomic/molecular Bose-Einstein condensation. In particular, two low excitation spectrum have been given at zero temperature, which show that the interaction between the hybrid atomic/molecular BEC increase one excitation mode, while they decrease another excitation mode. The mode $(0,1,0)$ comes from the density fluctuation of the condensate like vibrating oscillation, in this sense that the mode

$(0,1,0)$ increases near the Feshbach resonance indicates the presence of the Feshbach resonance enhance the density fluctuation like vibrating oscillation in atomic/molecular condensation system, whereas the breath mode (like breathing oscillation) $(1,0,0)$ decrease near the Feshbach resonance. Within the two-body collision regime, we show the atom number remained in BEC vs. the external magnetic field B , the result is in good agreement with the recent experiment. This work removes from consideration of the case at resonance, since at resonance the Bogoliubov approximation is not available (at resonance, there are few atoms in condensate). The contributions of the noncondensate atoms (molecules) to the excitation spectrum is also ignored (see eq.(9)). These need further investigations.

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Figure Captions

Fig.1: Excitation spectrum of Mode $(0,1,0)$ vs. number of atoms. Dotted and dashed line indicate those with and without Feshbach resonance, respectively.

Fig.2: Same as fig.1. But for mode $(1,0,0)$.

Fig.3: The number of atoms in condensate vs. magnetic field B .